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Common Coupled Fixed Point Result Using Common E.A Like Property inDislocated Quasi b-Metric Spaces

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Abstract

In this paper, we prove a common coupled fixed point theorem for four maps satisfying the common E.A.Like property in dislocated quasi b-metric spaces and obtain some corollaries from it. We also provide an example to illustrate our main theorem. Our results generalize and improve some results in existing literature.

Keywords:

E.A.Like property;

Dislocated quasi b-metric;

W-compatible maps.

Mathematics Subject Classification:54H25, 47H10.

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1. Introduction and Preliminaries

Banach [3] introduced the concept of fixed point theory in metric spaces. It is widely used in different areas like ordinary and partial differential equations, economics, convex optimization, electronic engineering and game theory etc. In recent years the Banach contraction principle is extended to generalized metrics such as quasi-metric, b-metric, partial metric, dislocated metric, dislocated quasi-metric and dislocated quasi b-metric etc.

Zeyada et.al [20] initiated the concept of dislocated quasi metric space and generalized the results of Hitzler and Seda [6] in dislocated quasi metric spaces.

The notion of b-metric space was introduced by Czerwic [5] in connection with some problems concerning with the convergence of non measurable functions with respect to measure.

Recently Klin-eam and Suanoom [8] introduced the concept of dislocated quasi b-metric spaces which generalize b-metric spaces [5] and quasi b-metric spaces [15] and proved some fixed point theorems in it by using cyclic contractions.

The authors [7,8,9,10,11,13,14,16,18,19,21,22] etc. obtained fixed and common fixed point theorems in dislocated quasi b-metric spaces using various contraction conditions for single and two maps.

K. Wadhwa et.al [17] introduced the definitions E.A.Like property in fuzzy metric spaces. The importance of E.A.Like property ensures that one does not require the completeness of the whole space or closedness of range subspaces and continuities of mappings. In this paper we adopt these definitions in dislocated quasi b-metric spaces as follows.

The purpose of this paper is to study the existence of common coupled fixed point for two pairs of mappings satisfying the common E.A.Like property in dislocated quasi b-metric spaces. Our results generalize and improve the Theorems 3.1 and 3.4 of [23].

First we recall some known definitions and lemmas. Throughout this paper, we assume that R^+ is the set of all non-negative real numbers.

Definition1.1 Let X be a non-empty set $, s \geq 1$ (a fixed real number) and $d: X \times X \to R^+$ be a function. Consider the following condition on d.

(1.1.1) $d(x,x) = 0, \forall x \epsilon X$

(1.1.2) $d(x,y) = d(y,x) = 0 \Rightarrow x = y, \forall x, y \epsilon X$

(1.1.3) $d(x,y) = d(y,x), , \forall x, y \epsilon X$

(1.1.4) $d(x,y) \le d(x,z) + d(z,y), \forall x,y,z \epsilon X$
(1.1.5) $d(x,y) \le s[d(x,z) + d(z,y)], \forall x,y,z \epsilon X$.

(i) If d satisfies (1.1.2), (1.1.3) and (1.1.4) then d is called a dislocated metric and (X,d) is called a dislocated metric space.

(ii) If d satisfies (1.1.1),(1.1.2) and (1.1.4) then d is called a quasi metric and (X,d) is called a quasi metric space.

(iii) If d satisfies (1.1.2) and (1.1.4) then d is called a dislocated quasi metric or dq-metric and (X,d) is called a dislocated quasi metric space.

(iv) If d satisfies (1.1.1),(1.1.2),(1.1.3) and (1.1.4) then d is called a metric and (X,d) is called a metric space.

(v) If d satisfies (1.1.1),(1.1.2),(1.1.3) and (1.1.5) then d is called a b-metric and (X,d) is called a b-metric space.

(vi) If d satisfies (1.1.2) and (1.1.5) then d is called a dislocated quasi b-metric and (X,d) is called a dislocated quasi b-metric space or dq b-metric space.

Definition 1.2 Let (X,d) be a dq b-metric space. A sequence $\{x_n\}$ in (X,d) is said to be

(i) dq b-convergent if there exists some point $x \epsilon X$ such that $\lim_{n\to\infty} d(x_n, x) = 0 = \lim_{n\to\infty} d(x, x_n)$.
In this case x is called a dq b-limit of $\{x_n\}$ and we write $x_n \to x as n \to \infty$.

(ii) Cauchy sequence if $\lim_{n,m\to\infty} d(x_n, x_m) = 0 = \lim_{n,m\to\infty} d(x_m, x_n)$.
The space (X,d) is called complete if every Cauchy sequence in X is dq b-convergent.
One can prove easily the following

Lemma 1.3 Let (X,d) be a dq b-metric space and $\{x_n\}$ be dq b-convergent to $x \epsilon X$ and $y \epsilon X$ be arbitrary.Then

$$\frac{1}{s} d(x,y) \le \liminf_{n\to\infty} d(x_n, y) \le \limsup_{n\to\infty} d(x_n, y) \le s d(x,y)$$

$$\frac{1}{s} d(y,x) \le \liminf_{n\to\infty} d(y, x_n) \le \limsup_{n\to\infty} d(y, x_n) \le s d(y,x).$$

The notion of coupled fixed point is introduced by Bhaskar and Lakshmikantham [4] and studied some fixed point theorems in partially ordered metric spaces. Later Lakshmikantham and Ciric [12] defined coupled coincidence point and common coupled fixed points for a pair of maps and Abbas et.al [2] introduced the notion of w-compatible mappings.

Definition 1.4 ([4]) Let X be a non-empty set .An element $(x,y) \epsilon X \times X$ is called a coupled fixed point of a mapping $F: X \times X \to X if x = F(x,y) and y = F(y,x)$.

Definition 1.5 ([12]) Let X be anon-empty set. An element $(x,y) \epsilon X \times X$ is called
(1) a coupled coincidence point of mappings $F: X \times X \to X$and $f: X \to$ X if fx = F(x,y) and fy = F(y,x).
(2) a common coupled fixed point of mappings $F: X \times X \to X$and $f: X \to$ Xif x = fx = F(x,y) and
y = fy = F(y,x).

Definition 1.6([2]) Let X be anon-empty set. The mappings $F: X \times X \to X$and $f: X \to$ X are called w-compatible if $f(F(x,y)) = F(fx, fy)$ and $f(F(y,x)) = F(fy, fx)$ whenever there exist $x, y \epsilon X$ such that fx = F(x,y) and fy = F(y,x).

Now we extend the definitions of E.A.Like property introduced by Wadhwaet.al [17] to dislocated quasi b-metric spaces as follows.

Definition 1.7 Let (X,d) be a dislocated quasi b-metric space and $F: X\times X \to X$and $S: X \to X$ be mappings. The pair (F,S) is said to satisfy common E.A.Like property if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} Sx_n = t$ and$\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} Sy_n = t^1$ for some $t, t^1 \epsilon$ S(X) or $F(X\times X)$.

Definition1.8 Let (X,d) be a dislocated quasi b-metric space and $F, G: X\times X \to X$and $S, T: X \to X$ be mappings. The pairs (F,S) and (G,T) are said to satisfy common E.A.Like property if there exist sequences $\{x_n\},\{y_n\},\{z_n\}$and$\{w_n\}$ in X such that

$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} G(z_n, w_n) = \lim_{n\to\infty} Tz_n = t$ and

$\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} G(w_n, z_n) = \lim_{n\to\infty} Tw_n = t^1$

for some $t, t^1 \epsilon S(X) \cap T(X)$ or $F(X\times X) \cap G(X\times X)$.

In this paper we utilize the following class of functions.

Definition1.9 Let**Φ** be the set of all functions $\varphi : R^+ \to R^+$ satisfying

(φ_1) : φ is continuous ,(φ_2) : φ is monotonically non-decreasing and (φ_3) : $\varphi(t) < t$for all $t > 0$.

2 Main Results

In this section, we give our main result.

Theorem2.1 Let (X,d) be a dislocated quasi b-metric space with $s \geq 1$ and $F, G : X \times X \to X$ and $S,T : X \to X$ be mappings satisfying

(2.1.1) $$d(F(x,y), G(u,v)) \leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Sx,Tu), d(Sy,Tv), \frac{1}{2}d(Sx,F(x,y)), \frac{1}{2}d(Sy,F(y,x)), \\ \frac{1}{2}d(Tu,G(u,v)), \frac{1}{2}d(Tv,G(v,u)), d(Sx,G(u,v)), \\ d(Sy,G(v,u)), d(Tu,F(x,y)), d(Tv,F(y,x)) \end{array}\right\}\right)$$

for all $x,y,u,v \epsilon X$, where $\varphi \epsilon \mathbf{\Phi}$,

(2.1.2) $$d(G(x,y), F(u,v)) \leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Tx,Su), d(Ty,Sv), \frac{1}{2}d(Tx,G(x,y)), \frac{1}{2}d(Ty,G(y,x)), \\ \frac{1}{2}d(Su,F(u,v)), \frac{1}{2}d(Sv,F(v,u)), d(Tx,F(u,v)), \\ d(Ty,F(v,u)), d(Su,G(x,y)), d(Sv,G(y,x)) \end{array}\right\}\right)$$

for all $x,y,u,v \epsilon X$, where $\varphi \epsilon \mathbf{\Phi}$,

(2.1.3) the pairs (F,S) and (G,T) satisfy common E.A.Like property and

(2.1.4) the pairs (F,S) and (G,T) are w-compatible.

Then F,G,S and T have a unique common coupled fixed point in $X \times X$.

Proof. Since (F,S) and (G,T) satisfy common E.A.Like property ,there exist sequences $\{x_n\},\{y_n\}$, $\{z_n\}$and$\{w_n\}$ in X such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} G(z_n, w_n) = \lim_{n\to\infty} Tz_n = t$ and

$\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} G(w_n, z_n) = \lim_{n\to\infty} Tw_n = t^1$

for some $t, t^1 \epsilon S(X) \cap T(X)$ or $F(X\times X) \cap G(X\times X)$.

Without loss of generality assume that $t, t^1 \epsilon S(X) \cap T(X)$.

Since $\lim_{n\to\infty} Sx_n = t \,\epsilon S(X)$ and $\lim_{n\to\infty} Sy_n = t^1 \epsilon S(X)$, there exist $u,v \epsilon X$ such that $t = Su$ and $t^1 = Sv$.

By Lemma 1.3, (2.1.1) ,(φ_2) and (φ_1),we have

$$\frac{1}{s}\, d(F(u,v), t) \leq \liminf_{n\to\infty}\, d(F(u,v), G(z_n, w_n))$$

$$\leq \liminf_{n\to\infty}\, \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Su,Tz_n), d(Sv,Tw_n), \frac{1}{2}d(Su,F(u,v)), \frac{1}{2}d(Sv,F(v,u)), \\ \frac{1}{2}d(Tz_n,G(z_n,w_n)), \frac{1}{2}d(Tw_n,G(w_n,z_n)), d(Su,G(z_n,w_n)), \\ d(Sv,G(w_n,z_n)), d(Tz_n,F(u,v)), d(Tw_n,F(v,u)) \end{array}\right\}\right)$$

$$\leq \liminf_{n\to\infty}\, \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(t,Tz_n), d(t^1,Tw_n), \frac{1}{2}d(t,F(u,v)), \frac{1}{2}d(t^1,F(v,u)), \\ s\max\{d(Tz_n,t), d(t,G(z_n,w_n))\}, \\ s\max\{d(Tw_n,t^1), d(t^1,G(w_n,z_n))\}, d(t,G(z_n,w_n)), \\ d(t^1,G(w_n,z_n)), d(Tz_n,F(u,v)), d(Tw_n,F(v,u)) \end{array}\right\}\right)$$

$$\leq \varphi\left(\frac{1}{s^2}\max\{0,0, d(t,F(u,v)), d(t^1,F(v,u)), 0,0,0,0, s\,d(t,F(u,v)), s\,d(t^1,F(v,u))\}\right)$$

$\leq \varphi\left(\frac{1}{s}\max\{d(t,F(u,v)),d(t^1,F(v,u)),d(F(u,v),t),d(F(v,u),t^1)\}\right)$ (1)

Similarly we can prove that

$\frac{1}{s}\ d(t,F(u,v)) \leq \varphi\left(\frac{1}{s}\max\{d(t,F(u,v)),d(t^1,F(v,u)),d(F(u,v),t),d(F(v,u),t^1)\}\right)$ (2)

$\frac{1}{s}\ d(t^1,F(v,u)) \leq \varphi\left(\frac{1}{s}\max\{d(t,F(u,v)),d(t^1,F(v,u)),d(F(u,v),t),d(F(v,u),t^1)\}\right)$ (3)

and

$\frac{1}{s}\ d(F(v,u),t^1) \leq \varphi\left(\frac{1}{s}\max\{d(t,F(u,v)),d(t^1,F(v,u)),d(F(u,v),t),d(F(v,u),t^1)\}\right)$ (4)

From (1),(2),(3) and (4) we have

$$\frac{1}{s}\max\left\{\begin{matrix} d(F(u,v),t),d(t,F(u,v)),\\ d(t^1,F(v,u)),d(F(v,u),t^1)\end{matrix}\right\} \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix} d(F(u,v),t),d(t,F(u,v)),\\ d(t^1,F(v,u)),d(F(v,u),t^1)\end{matrix}\right\}\right)$$

which in turn yields from (φ_3) and (1.1.2) that $F(u,v) = t$ and $F(v,u) = t^1$.

Thus Su = $F(u,v) = t$ and Sv = $F(v,u) = t^1$.

Since the pair (F, S) is w-compatible, we have

St = S (F(u,v)) = F (Su,Sv) =F (t,t^1) (5)

and St^1 = S (F(v,u)) = F (Sv,Su) = F (t^1 , t). (6)

Now consider

$$\frac{1}{s}\ d(St,t) = \frac{1}{s}\ d(F(t,t^1),t) \leq \lim_{n\to\infty}\inf\ d(F(t,t^1),G(z_n,w_n))$$

$$\leq \lim_{n\to\infty}\inf\ \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix} d(St,Tz_n),d(St^1,Tw_n),\frac{1}{2}d(St,St),\frac{1}{2}d(St^1,St^1),\\ \frac{1}{2}d(Tz_n,G(z_n,w_n)),\frac{1}{2}d(Tw_n,G(w_n,z_n)),d(St,G(z_n,w_n)),\\ d(St^1,G(w_n,z_n)),d(Tz_n,St),d(Tw_n,St^1)\end{matrix}\right\}\right)$$

$$\leq \lim_{n\to\infty}\inf\ \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix} d(St,Tz_n),d(St^1,Tw_n),s\max\{d(St,t),d(t,St)\},\\ s\max\{d(St^1,t^1),d(t^1,St^1)\},s\max\{d(Tz_n,t),d(t,G(z_n,w_n))\},\\ s\max\{d(Tw_n,t^1),d(t^1,G(w_n,z_n))\},d(St,G(z_n,w_n)),\\ d(St^1,G(w_n,z_n)),d(Tz_n,St),d(Tw_n,St^1)\end{matrix}\right\}\right)$$

$$\leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix} s\,d(St,t),s\,d(St^1,t^1),s\max\{d(St,t),d(t,St)\},\\ s\max\{d(St^1,t^1),d(t^1,St^1)\},0,0,s\,d(St,t),\\ s\,d(St^1,t^1),s\,d(t,St),s\,d(t^1,St^1)\end{matrix}\right\}\right)$$

$\leq \varphi\left(\frac{1}{s}\max\{d(St,t),d(t,St),d(t^1,St^1),d(St^1,t^1)\}\right)$ (7)

Similarly we have

$\frac{1}{s}\ d(t,St) \leq \varphi\left(\frac{1}{s}\max\{d(St,t),d(t,St),d(t^1,St^1),d(St^1,t^1)\}\right)$ (8)

$\frac{1}{s}\ d(t^1,St^1) \leq \varphi\left(\frac{1}{s}\max\{d(St,t),d(t,St),d(t^1,St^1),d(St^1,t^1)\}\right)$ (9)

and $\frac{1}{s}\ d(St^1,t^1) \leq \varphi\left(\frac{1}{s}\max\{d(St,t),d(t,St),d(t^1,St^1),d(St^1,t^1)\}\right)$ (10)

From (7),(8),(9) and (10), we have

$$\frac{1}{s}\max\left\{\begin{matrix} d(St,t),d(t,St),\\ d(t^1,St^1),d(St^1,t^1)\end{matrix}\right\} \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix} d(St,t),d(t,St),\\ d(t^1,St^1),d(St^1,t^1)\end{matrix}\right\}\right)$$

which in turn yields from (φ_3) and (1.1. 2) that St = t and St^1 = t^1.

Thus $F(t,t^1) = St = t$ (11)

and $F(t^1,t) = St^1 = t^1$ (12)

Since $\lim_{n\to\infty}$ Tz$_n$ = tϵ $T(X)$ and $\lim_{n\to\infty}$ Tw$_n$ = t^1 ϵ $T(X)$, there exist $\alpha,\beta\epsilon X$ such that $t = T\alpha$ and $t^1 = T\beta$.

Consider

$$\frac{1}{s}\ d(t,G(\alpha,\beta)) \leq \lim_{n\to\infty}\inf\ d(F(x_n,y_n),G(\alpha,\beta))$$

$$\leq \lim_{n\to\infty}\inf\ \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix} d(Sx_n,t),d(Sy_n,t^1),\frac{1}{2}d(Sx_n,F(x_n,y_n)),\frac{1}{2}d(Sy_n,F(y_n,x_n)),\\ \frac{1}{2}d(t,G(\alpha,\beta)),\frac{1}{2}d(t^1,G(\beta,\alpha)),d(Sx_n,G(\alpha,\beta)),\\ d(Sy_n,G(\beta,\alpha)),d(t,F(x_n,y_n)),d(t^1,F(y_n,x_n))\end{matrix}\right\}\right)$$

$$\leq \lim_{n\to\infty}\inf\ \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix} d(Sx_n,t),d(Sy_n,t^1),s\max\{d(Sx_n,t),d(t,F(x_n,y_n))\},\\ s\max\{d(Sy_n,t^1),d(t^1,F(y_n,x_n))\},d(t,G(\alpha,\beta)),\\ d(t^1,G(\beta,\alpha)),d(Sx_n,G(\alpha,\beta)),d(Sy_n,G(\beta,\alpha)),\\ d(t,F(x_n,y_n)),d(t^1,F(y_n,x_n))\end{matrix}\right\}\right)$$

$$\leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix}0,0,0,0,d\big(t,G(\alpha,\beta)\big),d\big(t^1,G(\beta,\alpha)\big),\\ s\,d\big(t,G(\alpha,\beta)\big),s\,d\big(t^1,G(\beta,\alpha)\big),0,0\end{matrix}\right\}\right)$$

$$\leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\}\right) \qquad (13)$$

Similarly we can prove that

$$\frac{1}{s}\,d\big(G(\alpha,\beta),t\big) \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\}\right) \qquad (14)$$

$$\frac{1}{s}\,d\big(t^1,G(\beta,\alpha)\big) \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\}\right) \qquad (15)$$

and

$$\frac{1}{s}\,d\big(G(\beta,\alpha),t^1\big) \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\}\right) \qquad (16)$$

From (13),(14),(15) and (16) we have

$$\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\} \leq \varphi\left(\frac{1}{s}\max\left\{\begin{matrix}d\big(t,G(\alpha,\beta)\big),d\big(G(\alpha,\beta),t\big),\\ d\big(t^1,G(\beta,\alpha)\big),d\big(G(\beta,\alpha),t^1\big)\end{matrix}\right\}\right)$$

which in turn yields from (φ_3) and (1.1.2) that $G(\alpha,\beta)=t$ and $G(\beta,\alpha)=t^1$.
Thus $T\alpha = G(\alpha,\beta)=t$ and $T\beta = G(\beta,\alpha)=t^1$.
Since the pair (G,T) is w-compatible, we have

$$Tt = T\big(G(\alpha,\beta)\big) = G(T\alpha,T\beta) = G(t,t^1) \qquad (17)$$

and

$$Tt^1 = T\big(G(\beta,\alpha)\big) = G(T\beta,T\alpha) = G(t^1,t). \qquad (18)$$

Using (2.1.1) and (φ_2), we have

$$d(t,Tt) = d(F(t,t^1),G(t,t^1))$$

$$\leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix}d(t,Tt),d(t^1,Tt^1),\frac{1}{2}d(t,t),\frac{1}{2}d(t^1,t^1),\\ \frac{1}{2}d(Tt,Tt),\frac{1}{2}d(Tt^1,Tt^1),d(t,Tt),\\ d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\end{matrix}\right\}\right)$$

$$\leq \varphi\left(\frac{1}{s^2}\max\left\{\begin{matrix}d(t,Tt),d(t^1,Tt^1),s\max\{d(t,Tt),d(Tt,t)\},\\ s\max\{d(t^1,Tt^1),d(Tt^1,t^1)\},s\max\{d(Tt,t),d(t,Tt)\},\\ s\max\{d(Tt^1,t^1),d(t^1,Tt^1)\},d(t,Tt),\\ d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\end{matrix}\right\}\right)$$

$$\leq \varphi\left(\max\{d(t,Tt),d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\}\right) \qquad (19)$$

Similarly we have

$$d(Tt,t) \leq \varphi\left(\max\{d(t,Tt),d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\}\right) \qquad (20)$$

$$d(t^1,Tt^1) \leq \varphi\left(\max\{d(t,Tt),d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\}\right) \qquad (21)$$

and $d(Tt^1,t^1) \leq \varphi\left(\max\{d(t,Tt),d(t^1,Tt^1),d(Tt,t),d(Tt^1,t^1)\}\right)$ (22)

From (19),(20),(21) and (22) we have

$$\max\left\{\begin{matrix}d(t,Tt),d(t^1,Tt^1),\\ d(Tt,t),d(Tt^1,t^1)\end{matrix}\right\} \leq \varphi\left(\max\left\{\begin{matrix}d(t,Tt),d(t^1,Tt^1),\\ d(Tt,t),d(Tt^1,t^1)\end{matrix}\right\}\right)$$

which in turn yields from (φ_3) and (1.1.2) that $Tt = t$ and $Tt^1 = t^1$.
Thus from (17) and (18) we have

$$G(t,t^1) = Tt = t \qquad (23)$$

and

$$G(t^1,t) = Tt^1 = t^1 \qquad (24)$$

From (11),(12),(23) and (24) it follows that (t,t^1) is a common coupled fixed point of F,G,S and T.
Uniqueness of common coupled fixed point of F,G,S and T follows easily from (2.1.1) and (2.1.2).
Now we give an example to illustrate our Main Theorem 2.1.

Example2.2 Let $X=[0,1]$ and $d(x,y)=|x-y|^2+|x|$. Then d is a dislocated quasi b-metric with s = 2.
Define F,G : $X\times X\to X$ and $S,T:X\to X$ by $F(x,y)=\frac{x^2+y^2}{32}$, $Sx=\frac{x^2}{2}$, $G(x,y)=\frac{x^2+y^2}{48}$ and $Tx=\frac{x^2}{3}$.
Let $\varphi:R^+\to R^+$ be defined by $\varphi(t)=\frac{t}{2}$, for $t\in R^+$.
Consider

$$d\big(F(x,y),G(u,v)\big) = d\left(\frac{x^2+y^2}{32},\frac{u^2+v^2}{48}\right) = \left|\frac{x^2+y^2}{32}-\frac{u^2+v^2}{48}\right|^2 + \left|\frac{x^2+y^2}{32}\right|$$

$$= \left|\frac{(3x^2-2u^2)+(3y^2-2v^2)}{(6)(16)}\right|^2 + \frac{x^2+y^2}{32}$$

$$= \frac{1}{(16)(16)}\left|\left(\frac{x^2}{2}-\frac{u^2}{3}\right)+\left(\frac{y^2}{2}-\frac{v^2}{3}\right)\right|^2+\frac{x^2+y^2}{32}$$
$$\le \frac{2}{(16)(16)}\left[\left|\frac{x^2}{2}-\frac{u^2}{3}\right|^2+\left|\frac{y^2}{2}-\frac{v^2}{3}\right|^2\right]+\frac{x^2+y^2}{32}$$
$$= \frac{1}{16}\left[\frac{1}{8}\left|\frac{x^2}{2}-\frac{u^2}{3}\right|^2+\frac{1}{8}\left|\frac{y^2}{2}-\frac{v^2}{3}\right|^2+\frac{x^2}{2}+\frac{y^2}{2}\right]$$
$$\le \frac{1}{16}\left[\left|\frac{x^2}{2}-\frac{u^2}{3}\right|^2+\frac{x^2}{2}+\left|\frac{y^2}{2}-\frac{v^2}{3}\right|^2+\frac{y^2}{2}\right]$$
$$= \frac{1}{16}[d(Sx,Tu)+d(Sy,Tv)]$$
$$\le \frac{1}{8}\max\{d(Sx,Tu),d(Sy,Tv)\}$$
$$= \frac{1}{2}\left(\frac{1}{s^2}\max\{d(Sx,Tu),d(Sy,Tv)\}\right)$$
$$\le \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Sx,Tu),d(Sy,Tv),\frac{1}{2}d(Sx,F(x,y)),\frac{1}{2}d(Sy,F(y,x)),\\ \frac{1}{2}d(Tu,G(u,v)),\frac{1}{2}d(Tv,G(v,u)),d(Sx,G(u,v)),\\ d(Sy,G(v,u)),d(Tu,F(x,y)),d(Tv,F(y,x)) \end{array}\right\}\right)$$

Similarly we can show that

$$d(G(x,y),F(u,v)) \le \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Tx,Su),d(Ty,Sv),\frac{1}{2}d(Tx,G(x,y)),\frac{1}{2}d(Ty,G(y,x)),\\ \frac{1}{2}d(Su,F(u,v)),\frac{1}{2}d(Sv,F(v,u)),d(Tx,F(u,v)),\\ d(Ty,F(v,u)),d(Su,G(x,y)),d(Sv,G(y,x)) \end{array}\right\}\right)$$

One can easily show that the pairs (F,S) and (G,T) satisfy the common E.A.Like property with $x_n=\frac{1}{n}, y_n=\frac{1}{n+1}, z_n=\frac{1}{2^n}$ and $w_n=\frac{1}{2^{n+1}}$ for n = 1,2,3,…,

Clearly the pairs (F,S) and (G,T) are w-compatible and (0,0) is the unique common coupled fixed point of F,G,S and T.

Corollary2.3 Let (X,d) be a dislocated quasi b-metric space with s ≥ 1 and and F : X × X→ X and S : X → X be mappings satisfying

(2.3.1)

$$d(F(x,y),F(u,v)) \le \varphi\left(\frac{1}{s^2}\max\left\{\begin{array}{c} d(Sx,Su),d(Sy,Sv),\frac{1}{2}d(Sx,F(x,y)),\frac{1}{2}d(Sy,F(y,x)),\\ \frac{1}{2}d(Su,F(u,v)),\frac{1}{2}d(Sv,F(v,u)),d(Sx,F(u,v)),\\ d(Sy,F(v,u)),d(Su,F(x,y)),d(Sv,F(y,x)) \end{array}\right\}\right)$$

for all x,y,u,vϵ X , where $\varphi\epsilon\mathbf{\Phi}$,

(2.3.2) the pair (F,S) satisfy the common E.A.Like property,

(2.3.3) the pair (F,S) is w-compatible.

Then F and S have a unique common coupled fixed point in X × X.

Corollary 2.4 Let(X,d) be a dislocated quasi metric space and f,g,S,T : X→ X be mappings satisfying

(2.4.1) $d(fx,gy) \le \varphi\left(\max\left\{d(Sx,Ty),\frac{1}{2}d(Sx,fx),\frac{1}{2}d(Ty,gy),d(Sx,gy),d(Ty,fx)\right\}\right)$

for all x,yϵ X , where $\varphi\epsilon\mathbf{\Phi}$,

(2.4.2) $d(gx,fy) \le \varphi\left(\max\left\{d(Tx,Sy),\frac{1}{2}d(Tx,gx),\frac{1}{2}d(Sy,fy),d(Tx,fy),d(Sy,gx)\right\}\right)$

for all x,yϵ X , where $\varphi\epsilon\mathbf{\Phi}$,

(2.4.3) the pairs (f,S) and (g,T) satisfy the common E.A.Like property

(2.4.4) the pairs (f,S) and (g,T) are weakly compatible.

Then f,g,S and T have a unique common fixed point in X.

Proof. It followsby taking s = 1and F (x,y) = fx and G(x,y) = gx in Theorem 2.1.

Corollary 2.5 Let (X,d) be a dislocated metric space and f,g,S,T : X→ X be mappings satisfying (2.4.3),(2.4.4) and

(2.5.1) $d(fx,gy) \le \varphi\left(\max\left\{d(Sx,Ty),\frac{1}{2}d(Sx,fx),\frac{1}{2}d(Ty,gy),d(Sx,gy),d(Ty,fx)\right\}\right)$

for all x,yϵ X , where $\varphi\epsilon\mathbf{\Phi}$, Then f,g,S and T have a unique common fixed point in X.

Corollary 2.6 Corollary 2.5 holds if (2.5.1) is replaced with

(2.6.1) $d(fx,gy) \leq k_1 d(Sx,Ty) + k_2 d(Sx,fx) + k_3 d(Ty,gy) + k_4 d(Sx,gy) + k_5 d(Ty,fx)$

for all x,yϵ X where k_1,k_2,k_3,k_4 and k_5 are non negative real numbers with $k_1+2k_2+2k_3+k_4+k_5 < 1$.

Remark: Corollary 2.6 is an improvement of Theorem 3.4 of [23].

Corollary 2.7 Let (X,d) be a dislocated quasi metric space and f, S : X→ X be mappings satisfying

(2.7.1) $d(fx,fy) \leq \varphi\left(\max\left\{d(Sx,Sy), \frac{1}{2}d(Sx,fx), \frac{1}{2}d(Sy,fy), d(Sx,fy), d(Sy,fx)\right\}\right)$

for all x,yϵ X , where $\varphi\epsilon$,**Φ**

(2.7.2) the pairs (f,S) satisfy the common E.A.Like property

(2.7.3) the pair (f,S) is weakly compatible.

Then f and S have a unique common fixed point in X.

Proof. It follows from Corollary 2.3 with s = 1 and F = f.

Remark : Corollary2.7 is an improvement of Theorem 3.1 of [23].

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